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Rational Spectra for Finite Hadamard Pairs and Bounded Spectral Sets on the real line

Xiaoye Fu

University of Central China Normal University

(joint work with Zi-Jian Song)

We establish that every finite spectral pair (A, Γ) with $A \subset \mathbb{Z}$, $\Gamma \subset \mathbb{R}/\mathbb{Z}$, and $0 \in \Gamma$ admits a rational spectrum, namely $\Gamma \subset \mathbb{Q}/\mathbb{Z}$. Our argument relies on a modulus rigidity theorem for generalized Vandermonde systems satisfying inverse-orthogonality identities, built from hyperbolic positive-definite kernels. Coupled with Galois conjugation and Kronecker's theorem, this rigidity property forces all associated exponential nodes to be roots of unity. As a consequential application, using the periodicity and fiberization of one-dimensional spectra, we show that every spectrum Λ of a bounded measurable spectral set $\Omega \subset \mathbb{R}$ with $|\Omega| = 1$ and $0 \in \Lambda$ is contained in $\mathbb{Q}$. This rational-spectrum result completes a chain of equivalences proven by Dutkay and Lai, which reduces the full one-dimensional Fuglede's conjecture to its finite cyclic analogues over $\mathbb{Z}_n$.