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Fourier edge complexity of complete bipartite graphs

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The eigenvalues of a graph's adjacency matrix are unchanged by relabeling its vertices. In contrast, a labeling of its N vertices by $\mathbb{Z}_N$ identifies the adjacency matrix with a function f on $\mathbb{Z}_N^2$ whose Fourier representation and compressibility can depend on the labeling. For $f \neq 0$, the Fourier ratio $\|\widehat{f}\|_{\ell^1}/\|\widehat{f}\|_{\ell^2}$ measures its Fourier complexity; its minimum over all vertex labelings is the graph's Fourier edge complexity. This quantity is motivated by signal recovery from partial observations.

The complete bipartite graph $K_{m,n}$ has two parts of sizes m and n, with every possible edge between the parts and none within either part. For $1 \leq m \leq n$, I will show how its labeling problem reduces to minimizing the Fourier ℓ^1-norm of indicators of m-element subsets of $\mathbb{Z}_{m+n}$. I will sketch the short proof of a lower bound using Fourier inversion and the triangle inequality, and state its equality characterization: equality holds exactly for labelings whose m-vertex part is labeled by a subgroup coset; such labelings exist precisely when $m \mid n$.

For $K_{3,n}$, this settles the case $3 \mid N$, where $N = n + 3$. When $3 \nmid N$, the minimizing three-element label sets are precisely three-term arithmetic progressions with invertible common difference. I will indicate how polynomial minorants establish this classification in the representative case of sufficiently large prime N, using the relation between even Fourier moments and additive energies. I will then state the complete classification theorem for optimal labelings of $K_{3,n}$ for every $n \geq 3$, with no primality restriction on $N = n+3$. I will conclude with examples of optimal label sets that are neither subgroup cosets nor arithmetic progressions, ending with questions about classification for larger parts.