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Weak Tiling, Keller's Theorem, and Fuglede's Conjecture for Three Squares

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Fuglede's conjecture asks whether a bounded measurable set admits an orthogonal basis of exponential functions if and only if it tiles the ambient Euclidean space by translations. In this talk, we study this connection for finite unions of non-overlapping unit cubes through the notion of weak tiling. Our main tool is a weak-tiling analogue of Keller's classical theorem, which imposes a structural restriction on the support of a weak tiling measure associated with the unit cube. We explain how this restriction yields geometric constraints on cube configurations. As an application, we prove that a union of three non-overlapping unit squares in the plane is spectral if and only if it tiles the plane by translations, without assuming that the squares are parallel. We also establish the equivalence of spectrality, weak tiling, and translational tiling for unions of two non-overlapping axis-parallel unit cubes in any dimension.