

HARMONIC AND SPECTRAL ANALYSIS

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Index estimates for sequences of harmonic maps

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(joint work with TOBIAS LAMM)

We consider sequences of harmonic maps from a closed two dimensional Riemann surfaces (M, h) into a closed Riemannian manifold (N, h) . They are critical points of the Dirichlet energy

$$E(u) = \frac{1}{2} \int_M |\nabla u|^2 dv_g.$$

We are interested in the study of upper and lower bounds of the index and the nullity for a sequences of harmonic maps with uniformly bounded Dirichlet energy.

The main difficulty stems from the fact that in the limit the sequence can develop finitely many bubbles. We obtain the index bounds by studying the limiting behavior of sequences of eigenfunctions of the linearized operator.

The key novelty of our approach is that we diagonalize the index form of the Dirichlet energy with respect to a bilinear form which varies with the sequence of harmonic maps and which helps us to show the convergence of the sequence of eigenfunctions on the weak limit, the bubbles and the intermediate neck regions.

If time permits I would like to talk about the “work in progress” extending the results to approximate harmonic maps.