Moments and Derivations on Hypergroups

Żywilla Fechner
Lodz University of Technology
(joint work with Eszter Gselmann and László Székelyhidi.)

The aim of this talk is to recall the moment problem, define (higher order) derivations and (generalized) moment generating functions in the hypergroup settings. The connection between these notions is given in the following theorem:

Theorem

Let X be a commutative hypergroup and r a positive integer. The family $(D_\alpha)_{\alpha \in \mathbb{N}^r}$ of self-mappings on $\mathcal{M}_c(X)$ is a continuous higher order derivation of order r if and only if there exists a generalized moment function sequence $(\varphi_\alpha)_{\alpha \in \mathbb{N}^r}$ of rank r such that

$$\langle D_\alpha \mu, f \rangle = \int_X f \cdot \varphi_\alpha \, d\mu \tag{1}$$

holds for each μ in $\mathcal{M}_c(X)$, f in $\mathcal{C}(X)$ and α in $\mathbb{N}^r$.

References

[1] W. R. Bloom and H. Heyer, *Harmonic analysis of probability measures on hypergroups*, de Gruyter Studies in Mathematics, vol. 20, Walter de Gruyter & Co., Berlin, 1995.

[2] Ż. F., E. Gselmann and L. Székelyhidi, *Moment functions and exponential monomials on commutative hypergroups*, Aequat. Math. 95, 1281–1290 (2021).

[3] Ż. F., E. Gselmann and L. Székelyhidi, *Moment Functions on Groups*, Results in Mathematics, 76 (4), art.171 (2021)

[4] Ż. F., E. Gselmann and L. Székelyhidi, *Endomorphisms and derivations of the measure algebra of commutative hypergroups*, online first, Indagationes Math.

[5] L. Székelyhidi, *Functional Equations on Hypergroups*, World Scientific Publishing Co. Pte. Ltd., New Jersey, London, 2012.