HARMONIC AND SPECTRAL ANALYSIS

International Zoom Conference
October 4–6 2023

General Information

The conference is held with the aid of the Zoom video conferencing program from **October 4**, 2023 to **October 6, 2023**. The participation is free but it is subjected to registration.

All conference **talks** are given **with the help of** the program **Zoom**, which enables sharing screen and also a whiteboard. The duration of every talk is at most 20 minutes, which is followed by a discussion of at most 5 minutes. There are no breaks between the talks within a session, therefore the schedule of the individual talks is only approximative. Speakers cannot inherit time from the previous talk. The time saved by shorter talks can be devoted to problems and remarks at the end of the session. If you have special wishes concerning the schedule, you are welcome to consult the conference secretary, *Eszter Gselmann* at the e-mail address *gselmann@science.unideb.hu*.

The host of the conference is the Department of Analysis of the University of Debrecen and the Alfréd Rényi Institute of Mathematics. The Organizing Committee consists of:

Prof. **László Székelyhidi**	(Chair of the Organizing Committee)
Prof. **Eszter Gselmann**	(Scientific Secretary)
Prof. **József Kolumbán**	(Scientific Secretary)

You can find the list of invited speakers, the list of the registered participants, the program and the abstracts in this booklet. Your questions may help the Organizing Committee to improve organization, so do not hesitate to contact the conference secretaries, *Eszter Gselmann* (gselmann@science.unideb.hu) and *József Kolumbán* (jkolumban@math.bme.hu).

The Organizing Committee would like to express its great appreciation to the **Alfréd Rényi Institute of Mathematics** that provided us Zoom access during the conference.

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List of Invited Speakers

1. Dutkay, Dorin *(University of Central Florida)*
   Title of the talk: Orthonormal bases and Parseval frames generated by Cuntz algebras and row co-isometries
   E-mail: Dorin.Dutkay@ucf.edu

2. Fechner, Żywilla *(Łódź University of Technology)*
   Title of the talk: Moments and Derivations on Hypergroups
   E-mail: zfechner@gmail.com

3. Grinberg, Eric *(University of Massachusetts Boston)*
   Title of the talk: The Fundamental Theorem of Trigonometry And Archimedes Angle Measure
   E-mail: eric.grinberg@umb.edu

4. Hirsch, Jonas *(University of Leipzig)*
   Title of the talk: Index estimates for sequences of harmonic maps
   E-mail: jonas.hirsch@math.uni-leipzig.de

5. Iosevich, Alex *(Universtiy of Rochester)*
   Title of the talk:
   E-amil: iosevich@math.rochester.edu

6. Ivković, Stefan *(Mathematical Institute of the Serbian Academy of Sciences and Arts)*
   Title of the talk: Topologically transitive operators on the space of Radon measures
   E-mail: stefan.iv10@outlook.com

7. Kiss, Gergely *(Alfréd Rényi Institute of Mathematics)*
   Title of the talk: Overview of Fuglede's conjecture on cyclic groups
   E-mail: kigergo57@gmail.com

8. Kolountzakis, Mihalis *(University of Crete)*
   Title of the talk: Connected examples for tiling and spectral problems
   E-mail: kolount@gmail.com

9. Kolumbán, József *(Budapest University of Technology and Economics)*
   Title of the talk: Constructing potential solutions to the Euler equations, with applications to some control problems in fluid-solid interactions
   E-mail: jkolumban@math.bme.hu

10. Kumar, Vishvesh *(Ghent University)*
    Title of the talk: L^p-L^q Fourier multipliers on hypergroups
    E-mail: vishveshmishra@gmail.com, Kumar.Vishvesh@UGent.be

11. Lᴇᴠ, Nir *(Bar-Ilan University)*
    Title of the talk: Non-periodic tilings of the real line by a function
    E-mail: levnir@gmail.com

12. Mᴀʟɪᴋɪᴏsɪs, Romanos Diogenes *(Aristotle University of Thessaloniki)*
    Title of the talk: A linear programming approach to Fuglede's conjecture in $\mathbb{Z}_p^3$
    E-mail: rwmanos@gmail.com

13. Mᴀᴛᴏʟᴄsɪ, Máté *(Rényi Institute of Mathematics and Budapest University of Technology and Economics)*
    Title of the talk: Tiling and weak tiling in $\mathbb{Z}_p^d$
    E-mail: matolcsi.mate@renyi.hu, matemato@gmail.com

14. Öᴢᴛᴏᴘ, Serap *(İstanbul University)*
    Title of the talk: Wiener type Orlicz Algebras
    E-mail: oztops@istanbul.edu.tr

15. Pᴜʟs, Michael *(John Jay College of Criminal Justice)*
    Title of the talk: The Pompeiu problem for locally compact groups
    E-mail: mpuls@jjay.cuny.edu

16. Sᴀᴡᴀɴᴏ, Yoshihiro *(Chuo University)*
    Title of the talk: Atomic decomposition of a subspace of BMO
    E-mail: yoshihiro-sawano@celery.ocn.ne.jp

17. Sᴜᴇᴜʀ, Franck *(University of Bordeaux)*
    Title of the talk: Observability cost of localized and microlocalized data
    E-mail: franck.sueur@math.u-bordeaux.fr

18. Vᴇɢᴀ, Luis *(Basque Center for Applied Mathematics)*
    Title of the talk:
    E-mail: lvega@bcamath.org

Program

October 4, Wednesday

13^{45}–14^{00} **Opening (László Székelyhidi)**

14^{00}–14^{20} Mihalis Kolountzakis, *Connected examples for tiling and spectral problems*

14^{30}–14^{50} Romanos Malikiosis, *A linear programming approach to Fuglede's conjecture in $\mathbb{Z}_p^3$*

15^{00}–15^{30} **Coffee Break**

15^{30}–16^{25} 2$^{\text{nd}}$ **Afternoon Session, Chairman: Mihalis Kolountzakis**

15^{30}–15^{50} Michael Puls, *The Pompeiu problem for locally compact groups*

16^{00}–16^{20} Dorin Dutkay, *Orthonormal bases and Parseval frames generated by Cuntz algebras and row co-isometries*

October 5, Thursday

9^{00}–9^{55}	1st **Morning Session, Chairperson: Eszter Gselmann**
9^{00}–9^{20}	Żywilla Fechner, *Moments and Derivations on Hypergroups*
9^{30}–9^{50}	Serap Öztop, *Wiener type Orlicz Algebras*
10^{00}–10^{30}	**Coffee Break**
10^{30}–11^{25}	2nd **Morning Session, Chairperson: Żywilla Fechner**
10^{30}–10^{50}	Visvesh Kumar, $\mathbb{L}^p - \mathbb{L}^q$ *Fourier multipliers on hypergroups*
11^{00}–11^{20}	Gergely Kiss, *Overview of Fuglede's conjecture on cyclic groups*
11^{30}–14^{00}	**Lunch**
14^{00}–14^{55}	1st **Afternoon Session, Chairman: Máté Matolcsi**
14^{00}–14^{20}	Nir Lev, *Non-periodic tilings of the real line by a function*
14^{30}–14^{50}	Eric Grinberg, *The Fundamental Theorem of Trigonometry And Archimedes Angle Measure*
15^{00}–15^{30}	**Coffee Break**
15^{30}–16^{25}	2nd **Afternoon Session, Chairman: Nir Lev**
15^{30}–15^{50}	Luis Vega,
16^{00}–16^{20}	Máté Matolcsi, *Tiling and weak tiling in $\mathbb{Z}_p^d$*

October 6, Friday

9^{00}–9^{55}	1st **Morning Session, Chairperson: Gergely Kiss**
9^{00}–9^{20}	József Kolumbán, *Constructing potential solutions to the Euler equations, with applications to some control problems in fluid-solid interactions*
9^{30}–9^{50}	Yoshihiro Sawano, *Atomic decomposition of a subspace of BMO*
10^{00}–10^{30}	**Coffee Break**
10^{30}–11^{25}	2nd **Morning Session, Chairman: László Székelyhidi**
10^{30}–10^{50}	Stefan Ivković, *Topologically transitive operators on the space of Radon measures*
11^{00}–11^{20}	**Problems and Remarks**
11^{30}–14^{00}	**Lunch**
14^{00}–14^{55}	1st **Afternoon Session, Chairman: József Kolumbán**
14^{00}–14^{20}	Frank Sueur, *Observability cost of localized and microlocalized data*
14^{30}–14^{50}	Jonas Hirsch, *Index estimates for sequences of harmonic maps*
15^{00}–15^{20}	**Coffee Break**
15^{30}–16^{20}	2nd **Afternoon Session, Chairman: László Székelyhidi**
15^{30}–15^{50}	Alex Iosevich,
16^{00}–16^{20}	**Closing (László Székelyhidi)**

Abstracts

Orthonormal bases and Parseval frames generated by Cuntz algebras and row co-isometries

DORIN DUTKAY

University of Central Florida

We will present a general procedure to obtain orthonormal bases by iterating Cuntz isometries, and to obtain Parseval frames by iterating row co-isometries. Such orthonormal bases include Fourier series, Walsh bases, Fourier series on Cantor-type measures and various generalizations.

Moments and Derivations on Hypergroups

ŻYWILLA FECHNER

Lodz University of Technology

(joint work with ESZTER GSELMANN and László Székelyhidi)

The aim of this talk is to recall the moment problem, define (higher order) derivations and (generalized) moment generating functions in the hypergroup settings. The connection between these notions is given in the following theorem:

Theorem

Let X be a commutative hypergroup and r a positive integer. The family $(D_\alpha)_{\alpha \in \mathbb{N}^r}$ of self-mappings on $\mathcal{M}_c(X)$ is a continuous higher order derivation of order r if and only if there exists a generalized moment function sequence $(\varphi_\alpha)_{\alpha \in \mathbb{N}^r}$ of rank r such that

$$\langle D_\alpha \mu, f \rangle = \int_X f \cdot \varphi_\alpha \, d\mu \tag{1}$$

holds for each μ in $\mathcal{M}_c(X)$, f in $C(X)$ and α in $\mathbb{N}^r$.

References

[1] W. R. Bloom and H. Heyer, *Harmonic analysis of probability measures on hypergroups*, de Gruyter Studies in Mathematics, vol. 20, Walter de Gruyter & Co., Berlin, 1995.

[2] Ż. F., E. Gselmann and L. Székelyhidi, *Moment functions and exponential monomials on commutative hypergroups*, Aequat. Math. 95, 1281–1290 (2021).

[3] Ż. F., E. Gselmann and L. Székelyhidi, *Moment Functions on Groups*, Results in Mathematics, 76 (4), art.171 (2021)

[4] Ż. F., E. Gselmann and L. Székelyhidi, *Endomorphisms and derivations of the measure algebra of commutative hypergroups*, online first, Indagationes Math.

[5] L. Székelyhidi, *Functional Equations on Hypergroups*, World Scientific Publishing Co. Pte. Ltd., New Jersey, London, 2012.

The Fundamental Theorem of Trigonometry

And Archimedes Angle Measure

Eric Grinberg

University of Massachusetts Boston

(joint work with Omar Hijab)

The concept of angle measure is foundational yet surprisingly little featured in elementary treatments. We develop it along lines implicit in Archimedes' work and show how it leads to the notion of complex numbers and the fundamental theorem of trigonometry. Thus these concepts join the integers and the completeness of the real numbers in having ancient roots.

Index estimates for sequences of harmonic maps

Jonas Hirsch

University of Leipzig

(joint work with Tobias Lamm)

We consider sequences of harmonic maps from a closed two dimensional Riemann surfaces (M, h) into a closed Riemannian manifold (N, h). They are critical points of the Dirichlet energy

$$E(u) = \frac{1}{2} \int_M |\nabla u|^2 \, dv_g.$$

We are interested in the study of upper and lower bounds of the index and the nullity for a sequences of harmonic maps with uniformly bounded Dirichlet energy.

The main difficulty stems from the fact that in the limit the sequence can develop finitely many bubbles. We obtain the index bounds by studying the limiting behavior of sequences of eigenfunctions of the linearized operator.

The key novelty of our approach is that we diagonalize the index form of the Dirichlet energy with respect to a bilinear form which varies with the sequence of harmonic maps and which helps us to show the convergence of the sequence of eigenfunctions on the weak limit, the bubbles and the intermediate neck regions.

If time permits I would like to talk about the "work in progress" extending the results to approximate harmonic maps.

Topologically transitive operators on the space of Radon measures

STEFAN IVKOVIĆ

Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia

In this talk, we characterize hypercyclic weighted composition operators acting on the C^*-algebra of all continuous complex valued functions on a locally compact, non-compact Hausdorff space. Moreover, we consider the dynamics of the adjoint of a weighted composition operator and we give necessary and sufficient conditions for this adjoint operator to be topologically transitive on the space of Radon measures on a locally compact, non-compact Hausdorff space. Finally, we provide sufficient conditions for this adjoint operator to be chaotic and we give concrete examples.

Overview of Fuglede's conjecture on cyclic groups

GERGELY KISS

Alfréd Rényi Institute of Mathematics

In the first part of my talk I will give a brief overview of the recent results connected to Fuglede's spectral set conjecture on cyclic groups. After that I will present my joint work with Thomas Fallon, Azita Mayeli and Gábor Somlai. We verified the spectral-tile direction of Fuglede's conjecture on cyclic groups of order p^2q^2r where p, q, r are primes and $r \geq p^2q^2$. I will discuss some new methods that we used in our proof.

Constructing potential solutions to the Euler equations, with applications to some control problems in fluid-solid interactions

JÓZSEF KOLUMBÁN

Budapest University of Technology and Economics

A broad class of solutions to the incompressible Euler equations in the absence of vorticity can be given by solving an appropriate Neumann-problem for the Laplacian. We show how to do this in the particular setting of fluid-solid interaction problems, obtaining an appropriate control function which acts on the outer part of the fluid domain and steers the solid in a desired manner.

Connected examples for tiling and spectral problems

MIHALIS KOLOUNTZAKIS

University of Crete

(joint work with RACHEL GREENFELD)

Recently Greenfeld and Tao found an example of a finite subset in $\mathbb{Z}^d$ (for some large d) which tiles $\mathbb{Z}^d$ by translations but only aperiodically, thus disproving the so-called *Periodic Tiling Conjecture* in high enough dimension.

Roughly 20 years ago the Fuglede (or Spectral set) conjecture was disproved by Tao (in the spectral implies tiling direction) and by Kolountzakis and Matolcsi (in the tiling implies spectral direction). In this problem the dimension d eventually got down to 3 for both directions.

In both these problems (aperiodicity and Fuglede conjecture) the examples found are highly dispersed subsets of $\mathbb{Z}^d$. In this work we show how to modify these examples to obtain (pathwise) *connected* subsets of $\mathbb{Z}^d$ as examples by increasing the dimension d.

L^p-L^q **Fourier multipliers on hypergroups**

VISHVESH KUMAR
Ghent University, Belgium
(joint work with MICHAEL RUZHANSKY)

In this talk, we will discuss Hörmander's L^p-L^q boundedness of Fourier multipliers on commutative hypergroups. We will also deduce the L^p-L^q boundedness of the spectral multipliers for the generalized radial Laplacian by examining our results on Chébli-Trimèche hypergroups. As a consequence, we obtain embedding theorems and time asymptotics for the L^p-L^q norms of the heat kernel for generalized radial Laplacian.

Non-periodic tilings of the real line by a function

Nir Lev

Bar-Ilan University

(joint work with Mihalis Kolountzakis)

By a *tiling* we mean a system $\{f(x - \lambda), \lambda \in \Lambda\}$ of translates of a function f that forms a partition of unity. The obvious examples of one-dimensional tilings are the periodic ones. I will present results and open problems on the existence of *non-periodic* tilings in one dimension.

A linear programming approach
to Fuglede's conjecture in $\mathbb{Z}_p^3$

ROMANOS DIOGENES MALIKIOSIS

Aristotle University of Thessaloniki

Delsarte's method on linear programming bounds is a very powerful tool which provides an upper bound on the size of a set A in an additive group G, whose difference set $A - A$ avoids a given set E. This tool may have limitations, but has been used successfully in various settings, most notably in the sphere packing problem in 8 and 24 dimensions.

Here, we will present an application of this method to Fuglede's conjecture in $G = \mathbb{Z}_p^3$, providing the following partial result: a set $A \subset G$ with cardinality

$$p(p - \sqrt{p} - \frac{1}{\sqrt{p}}) < |A| < p^2,$$

cannot be spectral.

Tiling and weak tiling in $\mathbb{Z}_p^d$

Máté Matolcsi

Rényi Institute of Mathematics and Budapest University of Technology and Economics

The notion of weak tiling was recently introduced by Nir Lev and Máté Matolcsi in connection with Fuglede's conjecture. In particular, all spectral sets tile their complements weakly. In this talk we initiate a systematic study of weak tiling in finite Abelian p-groups $\mathbb{Z}_p^d$. However, the question whether all spectral sets tile in $\mathbb{Z}_p^3$ remains open. The talk is based on joint work with G. Kiss, D. Matolcsi, G. Somlai.

Wiener type Orlicz Algebras

Serap Öztop

İstanbul University

(joint work with Büşra Arıs)

Let G be a locally compact group, Φ_1, Φ_2 be Young functions and ω be a moderate weight function on G. We introduce the weighted Orlicz amalgam spaces $W(L^{\Phi_1}(G), L^{\Phi_2}_\omega(G))$ defined on G, where the local component is the Orlicz space $L^{\Phi_1}(G)$ and the global component is the weighted Orlicz space $L^{\Phi_2}_\omega(G)$. We present conditions for an Orlicz amalgam space to be a Banach algebra.

The Pompeiu problem for locally compact groups

MICHAEL PULS

John Jay College of the City University of New York

Let $n \geq 2$ and let K be a compact subset of $\mathbb{R}^n$ with positive Lebesgue measure. The classical Pompeiu problem asks the following: Is $f = 0$ the only continuous function on $\mathbb{R}^n$ that satisfies

$$\int_{\sigma(K)} f \, dx = 0$$

for all rigid motions σ?

In this talk we will discuss this problem in the setting of locally compact groups G, with continuous functions replaced by $L^2(G)$ and rigid motions replaced by translations. Unlike $\mathbb{R}^n$, G might not be abelian, so we will focus on two-sided translations. We will give results for various classes of groups, specifically type I groups and discrete groups.

Atomic decomposition of a subspace of BMO

SAWANO YOSHIHIRO
Chuo University

(joint work with DACHUN YANG AND WEN YUAN)

The goal of my talk is to introduce a new decomposition of a subspace of BMO. This is a continuation of what I have been doing for other function spaces. Around 1990, Frazier and Jawerth introduced the technique of obtaining non-smooth atoms from wavelet decomposition. This idea was revisited by Grafakos in his text book Modern Harnomic Analysis. This technique together with the reexamination of the atomic decomposition of Hardy spaces with variable exponents brought out a new technique to decompose functions in other spaces such as Triebel-Lizorkin-Morrey spaces. My talk reports an advancement in this direction.

Observability cost of localized and microlocalized data

Franck Sueur

University of Bordeaux

In this talk, I will expose some recent works obtained together with Roberta Bianchini (Rome) and Vincent Laheurte (Bordeaux) on the observability cost of linear first order evolution systems for initial data which are localized in the physical space or microlocalized in the phase space.