HARMONIC AND SPECTRAL ANALYSIS

International Zoom Conference

May 31 – June 2 2021

General Information

The conference is held with the aid of the Zoom video conferencing program from **May 31**, 2021 to **June 2, 2021**. The participation is free but it is subjected to registration.

All conference **talks** are given **with the help of** the program **Zoom**, which enables sharing screen and also a whiteboard. The duration of every talk is at most 20 minutes, which is followed by a discussion of at most 5 minutes. There are no breaks between the talks within a session, therefore the schedule of the individual talks is only approximative. Speakers cannot inherit time from the previous talk. The time saved by shorter talks can be devoted to problems and remarks at the end of the session. If you have special wishes concerning the schedule, you are welcome to consult the conference secretary, *Eszter Gselmann* at the e-mail address gselmann@science.unideb.hu.

The host of the conference is the Department of Analysis of the University of Debrecen and the Alfréd Rényi Institute of Mathematics. The Organizing Committee consists of:

Prof.	**László Székelyhidi**	(Chair of the Organizing Committee)
Dr.	**Eszter Gselmann**	(Scientific Secretary)
Dr.	**Gergely Kiss**	(Scientific Secretary)

You can find the list of invited speakers, the list of the registered participants, the programme and the abstracts in this booklet. Your questions may help the Organizing Committee to improve organization, so do not hesitate to contact the conference secretaries, *Eszter Gselmann* (gselmann@science.unideb.hu) and *Gergely Kiss* (kigergo57@gmail.com).

More information about the Harmonic and Spectral Analysis conferences can be found at our webpage: http://mathspectral.hu/.

The Organizing Committee would like to express its great appreciation to the **Alfréd Rényi Institute of Mathematics** that provided us Zoom access during the conference.

Project no. K134191 supporting E. Gselmann and L. Székelyhidi, has been implemented by the support provided from the National Research, Development and Innovation Fund of Hungary, financed under the K_20 funding scheme.

List of Invited Speakers

1. Dutkay, Dorin *(University of Central Florida, Florida, USA)*
   Title of the talk: Fourier series on singular measures
   E-mail: Dorin.Dutkay@ucf.edu

2. Fallon, Thoman *(The Graduate Center at the City University of New York, New York, USA)*
   Title of the talk: The Fuglede conjecture holds in $\mathbb{Z}_{p^2} \times \mathbb{Z}_{q^2}$
   E-mail: tfallon@gradcenter.cuny.edu

3. Fechner, Żywilla *(Łódź University of Technology, Łódź, Poland)*
   Title of the talk: Moment functions and exponential monomials on commutative hypergroups
   E-mail: zfechner@gmail.com

4. Greenfeld, Rachel *(University of California, Los Angeles, USA)*
   Title of the talk: Translational tilings in lattices
   E-mail: greenfeld@math.ucla.edu

5. Grinberg, Eric *(University of Massachusetts Boston, Boston, USA)*
   Title of the talk: Invariant function algebras on Hermitian symmetric domains with boundary
   E-mail: eric.grinberg@umb.edu

6. Iosevich, Alexander *(Universtiy of Rochester, New York, USA)*
   Title of the talk: On the Euclidean distance graph
   E-mail: iosevich@math.rochester.edu

7. Ivković, Stefan *(Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia)*
   Title of the talk: Semi-Fredholm operators on Hilbert C*-modules
   E-mail: stefan.iv10@outlook.com

8. Kolountzakis, Mihalis *(University of Crete, Crete, Greece)*
   Title of the talk: Finite variations on the Steinhaus tiling problem
   E-mail: kolount@gmail.com

9. Kumar, Vishvesh *(Ghent University, Ghent, Belgium)*
   Title of the talk: Hardy–Littlewood inequality and $L^p - L^q$ Fourier multipliers on compact hypergroups
   E-mail: vishveshmishra@gmail.com, Kumar.Vishvesh@UGent.be

10. Laczkovich, Miklós *(Eötvös Loránd University, Budapest, Hungary)*
    Title of the talk: Translation invariant linear spaces of polynomials and a problem posed by László Székelyhidi
    E-mail: miklos.laczkovich@gmail.com

11. Lev, Nir *(Bar-Ilan University, Ramat Gan, Izrael)*
    Title of the talk: Spectrality of polytopes and equidecomposability by translations
    E-mail: levnir@gmail.com

12. Linnell, Peter A. *(Virginia Tech, Virginia, USA)*
    Title of the talk: The discrete Pompeiu problem
    E-mail: plinnell@math.vt.edu

13. Machado, Fabrício Caluza *(University of São Paulo, São Paulo, Brazil)*
    Title of the talk: The null set of a polytope, and the Pompeiu property for polytopes
    E-mail: fabcm1@gmail.com

14. Matolcsi, Máté *(Budapest University of Technology and Economins and Alfréd Rényi Institute of Mathematics, Budapest, Hungary)*
    Title of the talk: Fuglede's conjecture for convex domains
    E-mail: matolcsi.mate@renyi.hu, matemato@gmail.com

15. Mattheus, Sam *(Vrije Universiteit Brussel, Brussels, Belgium)*
    Title of the talk: Fuglede's conjecture in elementary abelian groups
    E-mail: sam.mattheus@vub.ac.be

16. Mayeli, Azita *(The Graduate Center and Queensborough of the City University of New York, New York, USA)*
    Title of the talk: ϕ-approximate orthogonality on the unit ball
    E-mail: amayeli@gc.cuny.edu

17. Ramm, Alexander G. *(Kansas State University, Manhattan, USA)*
    Title of the talk: Symmetry problems in PDE
    E-mail: ramm@ksu.edu

18. Shi, Ruxi *(Mathematical Institute of the Polish Academy of Sciences, Warsaw, Poland)*
    Title of the talk: Spectral measures on groups with one prime factor
    E-mail: rshi@impan.pl

19. Tabatabaie, Mohammad S. *(University of Qom, Qom, Iran)*
    Title of the talk: Some properties of weighted Orlicz algebras
    E-mail: sm-tabatabaei@qom.ac.ir

20. Wilkens, Bettina *(University of Namibia, Windhoek, Namibia)*
    Title of the talk: Local spectral synthesis from a ring-theoretic perspective
    E-mail: bwlklk@gmail.com

List of Participants

1. Adriaensen, Sam *(Vrije Universiteit Brussel, Brussels, Belgium)*
2. Agbeko, Kwami *(University of Miskolc, Miskolc, Hungary)*
3. Almira, José María *(University of Murcia, Murcia, Spain)*
4. De Beule, Jan *(Vrije Universiteit Brussel, Brussels, Belgium)*
5. Forti, Gian Luigi *(University of Milan, Milan, Italy)*
6. Fechner, Włodzimierz *(Łódź University of Technology, Łódź, Poland)*
7. Gilányi, Attila *(University of Debrecen, Debrecen, Hungary)*
8. Gselmann, Eszter *(University of Debrecen, Debrecen, Hungary)*
9. Hui, Shui Wing David *(Hong Kong University of Science and Technology, Hong Kong)*
10. Kiss, Gergely *(Alfréd Rényi Institute of Mathematics, Budapest, Hungary)*
11. Kiss, Tibor *(University of Debrecen, Debrecen, Hungary)*
12. Ódor, Tibor *(University of Szeged, Szeged, Hungary and Alfréd Rényi Institute of Mathematics, Budapest, Hungary)*
13. Oliveira, Artur André Almeida de Macedo *(University of São Paulo, São Paulo, Brazil)*
14. Öztop, Serap *(Istanbul University, Istanbul, Turkey)*
15. Kurusa, Árpád *(University of Szeged, Szeged, Hungary)*
16. Papageorgiou, Effie *(University of Crete, Heraklion, Greece)*
17. Puls, Michael *(John Jay College of Criminal Justice, New York, USA)*
18. Révész, Szilárd *(Alfréd Rényi Institute of Mathematics, Budapest, Hungary)*
19. Sablik, Maciej *(University of Silesia in Katowice, Katowice, Poland)*
20. Schwaiger, Jens *(University of Graz, Graz, Austria)*
21. Sikorska, Justyna *(University of Silesia in Katowice, Katowice, Poland)*
22. Singh, Ajit Iqbal *(Indian National Science Academy, New Delhi, India)*
23. Somlai, Gábor *(Eötvös Loránd University, Budapest, Hungary)*

24. Syga, Emanuel *(Tel Aviv University, Tel Aviv, Israel)*
25. Székelyhidi, László *(University of Debrecen, Debrecen, Hungary)*
26. Tao, Terence *(University of California, Los Angeles, USA)*
27. Yan-Song, Fu *(China University of Mining and Technology, Beijing, China)*
28. Vati, Kedumetse *(Shanghai Jiao Tong University, Shanghai, China)*
29. Vértesi, Péter *(Alfréd Rényi Institute of Mathematics, Budapest, Hungary)*
30. Zheng, Yizhong *(City University of New York, New York, USA)*

Program

May 31, Monday

14^{00}–16^{00}	1st **Afternoon Session, Chairman: László Székelyhidi**
14^{00}–14^{20}	**Opening (László Székelyhidi)**
14^{30}–14^{50}	**Fabrício Caluza Machado**, *The null set of a polytope, and the Pompeiu property for polytopes*
15^{00}–15^{20}	**Alexander Ramm**, *Symmetry problems in PDE*
15^{30}–15^{50}	**Peter A. Linnell**, *The discrete Pompeiu problem*
16^{00}–16^{30}	**Coffee Break**
16^{30}–17^{30}	2nd **Afternoon Session, Chairman: Michael Puls**
16^{30}–16^{50}	**Eric Grinberg**, *Invariant function algebras on Hermitian symmetric domains with boundary*
17^{00}–17^{20}	**Rachel Greenfeld**, *Translational tilings in lattices*

June 1, Tuesday

11^{00}–12^{30}	1st **Morning Session, Chairperson: Żywilla Fechner**
11^{00}–11^{20}	**Mohammad S. Tabatabaie**, *Some properties of weighted Orlicz algebras*
11^{30}–11^{50}	**Vishvesh Kumar**, *Hardy-Littlewood inequality and $L^p - L^q$ Fourier multipliers on compact hypergroups*
12^{00}–12^{20}	**Miklós Laczkovich**, *Translation invariant linear spaces of polynomials and a problem posed by László Székelyhidi*
12^{30}–14^{20}	**Lunch Break**
14^{30}–16^{00}	1st **Afternoon Session, Chairman: Szilárd Révész**
14^{30}–14^{50}	**Máté Matolcsi**, *Fuglede's conjecture for convex domains*
15^{00}–15^{20}	**Nir Lev**, *Spectrality of polytopes and equidecomposability by translations*
15^{30}–15^{50}	**Mihalis Kolountzakis**, *Finite variations on the Steinhaus tiling problem*
16^{00}–16^{30}	**Problems and Remarks**
16^{30}–17^{30}	2nd **Afternoon Session, Chairman: Mihalis Kolountzakis**
16^{30}–16^{50}	**Alex Iosevich**, *On the Euclidean distance graph*
17^{00}–17^{20}	**Azita Mayeli**, *ϕ-approximate orthogonality on the unit ball*

June 2, Wednesday

11^{00}–12^{30}	1st **Morning Session, Chairperson: Eszter Gselmann**
11^{00}–11^{20}	**Żywilla Fechner**, *Moment functions and exponential monomials on commutative hypergroups*
11^{30}–11^{50}	**Stefan Ivković**, *Semi-Fredholm operators on Hilbert C^*-modules*
12^{00}–12^{20}	**Bettina Wilkens**, *Local spectral synthesis from a ring-theoretic perspective*
12^{30}–14^{20}	**Lunch Break**
14^{30}–16^{00}	1st **Afternoon Session, Chairman: Gergely Kiss**
14^{30}–14^{50}	**Sam Mattheus**, *Fuglede's conjecture in elementary abelian groups*
15^{00}–15^{20}	**Ruxi Shi**, *Spectral measures on groups with one prime factor*
15^{30}–15^{50}	**Thomas Fallon**, *The Fuglede conjecture holds in $\mathbb{Z}_{p^2} \times \mathbb{Z}_{q^2}$*
16^{00}–16^{30}	**Coffee Break**
16^{30}–17^{30}	2nd **Afternoon Session, Chairperson: Azita Mayeli**
16^{30}–16^{50}	**Dorin Dutkay**, *Fourier series on singular measures*
17^{00}–17^{20}	**Closing (László Székelyhidi)**

Abstracts

Fourier series on singular measures

DORIN DUTKAY
University of Central Florida

In connection to the Fuglede conjecture, Jorgensen and Pedersen discovered in 1996 that certain Cantor-type measures admit complete orthogonal Fourier series, that is orthonormal bases of exponential functions for their L^2 space. In this talk we will present some recent results in this area, convergence properties of Fourier series on fractal measures, connections between the Beurling dimension of the spectrum and the Hausdorff dimension of the support, tiling and spectral properties, extensions and generalizations.

The Fuglede conjecture holds in $\mathbb{Z}_p^2 \times \mathbb{Z}_q^2$

Thomas Fallon
CUNY Graduate Center
(joint work with Gergely Kiss and Gábor Somlai)

We examine the Fuglede's Conjecture, which in a given context states that spectral sets and tiling sets are the same sets. This problem was first proposed in the 1970s over $\mathbb{R}^d$ and there has been interest in the discrete version of the problem since the 2000s, as counterexamples there can be lifted to the euclidean case. Techniques of discrete geometry are used to examine tiling and spectral sets in this setting, allowing us to prove that this holds on the family of groups of the form $\mathbb{Z}_p^2 \times \mathbb{Z}_q^2 \cong \mathbb{Z}_{pq}^2$.

Moment functions and exponential monomials on commutative hypergroups

ŻYWILLA FECHNER
Lodz University of Technology
(joint work with ESZTER GSELMANN AND LÁSZLÓ SZÉKELYHIDI)

In this talk we are going to discuss a connection between moment functions and exponential monomials on commutative hypergroups. At the beginning we are going to give a brief motivation of the problem. In particular, we formalize the notions of an exponential monomial and moment function. We want to find conditions under which exponential monomials can be expressed in terms of generalized moment functions. We present one of possible conditions and discuss some possible ways for further research.

REFERENCE

[1] Ż. F., Eszter Gselmann, László Székelyhidi, Moment functions and exponential monomials on commutative hypergroups, submitted

Translational tilings in lattices

RACHEL GREENFELD
University of California
(joint work with TERENCE TAO)

Let F be a finite subset of $\mathbb{Z}^d$. We say that F is a *translational tile* of $\mathbb{Z}^d$ if it is possible to cover $\mathbb{Z}^d$ by translates of F without any overlaps. The well-known periodic tiling conjecture suggests that any translational tile admits at least one periodic tiling. In the talk, we will motivate and discuss the study of this conjecture. We will also present some new results, joint with Terence Tao, on the structure of translational tilings in lattices and introduce some applications.

Invariant function algebras on Hermitian symmetric domains with boundary

ERIC GRINBERG
University of Massachusetts Boston

We consider the algebra of continuous functions on a Hermitian symmetric domain with boundary and aim to classify group invariant subalgebras. The meta-theorem is that all such subalgebras are encoded by complex structure in the interior and by topological stratification of the boundary. The heuristic approach is to use the action of the group of symmetries to show that the sets of anti-symmetry of a candidate invariant algebra are few and readily classified.

On the Euclidean distance graph

Alex Iosevich
University of Rochester

Let E be a compact subset of $\mathbb{R}^d$, $d \geq 2$ of a given Hausdorff dimension. Let $\mathcal{G}_t(E)$ be a graph where the vertices are the points of E and two vertices are connected by an edge if $|x-y| = t$. A related problem in the realm of subsets of $\mathbb{R}^d$ of positive upper Lebesgue density was and continues to be explored by Furstenberg, Katznelson, Lyall, Magyar, Weiss, Ziegler, and others. We are going to survey the current state of knowledge of the structure of this graph and describe some recent results, co-authored with Greenleaf, Magyar, Mkrtchyan, Shen and others.

Semi-Fredholm operators on Hilbert C^*-modules

STEFAN IVKOVIĆ
The Mathematical institute of the Serbian Academy of Sciences and Arts

Based on the definition of C^*-Fredholm operator on the standard module given by Mishchenko and Fomenko we define semi-C*-Fredholm operators and semi-C^*-Weyl operators on Hilbert C^*-modules. We provide generalization in this context of various results from the classical semi-Fredholm theory. For instance we prove that these new classes of operators and their various subclasses are open in the norm topology, closed under multiplication, invariant under compact perturbations and many other results generalizing their classical counterparts. Both adjointable and non-adjointable operators are considered. In the special case of operators over W^*-algebras we obtain also generalization of Schechter-Lebow characterization and punctured neighbourhood theorem. Moreover, by considering exact sequences of regular operators we obtain generalization in the setting of operators over C^*-algebras of the results by Djorjević on generalized Weyl operators, of index theorem etc. Finally, we consider upper triangular 2 by 2 operator matrices over C^*-algebras and describe the relationship of semi-C^*-Fredholmness of these matirces and their diagonal entries. However, the main topic of this lecture will be generalized spectral semi-Fredholm theory in the setting of new spectra in C^*-algebras. This is a continuation of the lecture from previous year on the conference „Harmonic and Spectral Analysis“, when the generalized spectra in C^*-algebra of operators on Hilbert C^*-modules were introduced and the description of such spectra of shift operators, unitary, self-adjoint and normal operators on the standard C^*-module has been given. Various subclasses of semi-C^*-Fredholm operators induce different generalized spectra in C^*-algebras of operators on Hilbert C^*-modules. On this lecture we will provide generalizations in this setting of the results from the classical spectral semi-Fredholm theory.

Finite variations on the Steinhaus tiling problem

MIHALIS KOLOUNTZAKIS
University of Crete
(joint work with EFFIE PAPAGEORGIOU)

This is a talk about an object tiling simultaneously with many different groups. The original Steinhaus Tiling Problem asks for a subset of the plane that tiles with all rotations of $\mathbb{Z}^2$. One variation of this is to ask for a set that tiles with finitely many rotations. Another is to ask for a function (not necessarily an indicator function) that can tile with many (or all) rotations of $\mathbb{Z}^2$. Can such a function have bounded support? If yes, how large must the diameter be? How large must the area of the support be? What if one tries to tile an abelian group G? If G_1, G_2 are subgroups of G can one find a subset of G that tiles when translated by G_1 and also when translated by G_2? If we cannot find a set can we find a function then? How "large" must the support of this function be in this case? We will show some results, some old and some new, and many more questions.

Hardy-Littlewood inequality and L^p-L^q Fourier multipliers on compact hypergroups

VISHVESH KUMAR
Ghent University
(joint work with MICHAEL RUZHANSKY)

In this talk, we will discuss the inequalities devoted to the comparison between the norm of a function on a compact hypergroup and the norm of its Fourier coefficients. We prove the classical Paley inequality in the setting of compact hypergroups which further gives the Hardy-Littlewood and Hausdorff-Young-Paley inequalities in the noncommutative context. We establish Hörmander's L^p-L^q Fourier multiplier theorem on compact hypergroups for $1 < p \leq 2 \leq q < \infty$ as an application of Hausdorff-Young-Paley inequality.

Translational invariant linear spaces of polynomials and a problem posed by László Székelyhidi

MIKLÓS LACZKOVICH
Eötvös Loránd University
(joint work with GERGELY KISS)

We give a description of all translational invariant linear subspaces ('modules') of $\mathbb{C}[x, y]$, and show that not every module is relatively closed in $\mathbb{C}[x, y]$.

Spectrality of polytopes and equidecomposability by translations

NIR LEV
Bar-Ilan University
(joint work with BOCHEN LIU)

Fuglede conjectured that a domain $A \subset \mathbb{R}^d$ is spectral (that is, A admits an orthogonal basis of exponential functions) if and only if one can tile the space by translated copies of A. Matolcsi and the speaker recently proved this conjecture for convex domains, but for non-convex ones the conjecture was disproved by Tao. However I will present a result joint with Bochen Liu, which implies that if a non-convex polytope $A \subset \mathbb{R}^d$ is spectral then A can be dissected into a finite number of smaller polytopes, which can be rearranged using translations to form a cube.

The discrete Pompeiu problem

Peter Linnell
Virginia Tech
(joint work with Mike Puls)

Let K be a compact subset of $\mathbb{R}^n$ with nonzero Lebesgue measure. The Pompeiu problem asks if $f = 0$ is the only continuous function such that $\int_{\sigma(K)} f\,dx = 0$ for all rigid motions σ of $\mathbb{R}^n$. We will consider a version of the Pompeiu problem for discrete groups. We shall also describe $\mathcal{U}(G)$ and its role in this problem.

The null set of a polytope, and the Pompeiu property for polytopes

Fabrício Caluza Machado
University of São Paulo
(joint work with Sinai Robins)

A bounded domain $P \subset \mathbb{R}^d$ has the Pompeiu property if the values $\int_{\sigma(P)} f(x)\mathrm{d}x$ over all rigid motions $\sigma \in M(d)$ uniquely determine $f \in C(\mathbb{R}^d)$. The Fourier-Laplace transform of a set P is the funtion $\widehat{1}_P \colon \mathbb{C}^d \to \mathbb{C}$, $\widehat{1}_P(z) = \int_P e^{-2\pi i\langle x,z\rangle}\mathrm{d}x$ and the null set is $N(P) = \left\{z \in \mathbb{C}^d : \hat{1}_P(z) = 0\right\}$. A classical result from Brown, Schreiber, and Taylor (1973) states that a set P does not have the Pompeiu property if and only if there exists $\alpha \in \mathbb{C}\backslash\{0\}$ such that $\left\{z \in \mathbb{C}^d : z_1^2 + \cdots z_d = \alpha\right\}$ is contained in $N(P)$. In this work we show that the null set of a polytope does not contain (almost all) circles and hence have the Pompeiu property. The proof is relatively simple and uses properties of Bessel functions together with the Brion-Barvinok theorem, which gives a concrete formulation for the Fourier-Laplace transform of a polytope. arXiv preprint: 2104.01957.

Fuglede's conjecture for convex domains

MÁTÉ MATOLCSI
Budapest University of Technology and Economics
and
Alfréd Rényi Institute of Mathematics

In a recent joint work with Nir Lev we proved that Fuglede's conjecture holds for all convex domains in all dimensions. That is, we proved that a convex domain A in $\mathbb{R}^n$ tiles the space by translation if and only if A is spectral (i.e. $\mathbb{L}^2(A)$ has an orthogonal basis of exponentials).

Fuglede's conjecture in elementary abelian groups

SAM MATTHEUS
Vrije Universiteit

Fuglede's conjecture, dating back to 1974, roughly states that tiling sets and spectral sets in $\mathbb{R}^d$ are one and the same. This conjecture has been extended to arbitrary abelian groups, motivated by the fact that the counterexample in $\mathbb{R}^5$ found by Tao originates from a counterexample in the abelian group $(\mathbb{Z}/3\mathbb{Z})^5$. It is this setting we will focus on.

We will discuss Fuglede's conjecture in the elementary abelian groups $(\mathbb{Z}/p\mathbb{Z})^d$ from a geometrical perspective, review its current state, and consider related conjectures in finite geometry which might be unknown, yet interesting, to researchers in harmonic analysis.

ϕ-approximate orthogonality on the unit ball

AZITA MAYELI
City University of New York
The Graduate Center and Queensborough

Assume that $\Omega \subset \mathbb{R}^d$ is a Lebesgue measurable domain of finite measure, and $\phi : [0, \infty) \to [0, \infty)$ is a function with $\phi(\xi) \to 0$ as $|\xi| \to \infty$. If for $a \neq a'$

$$|\widehat{\chi_\Omega}(a - a')| \leq \phi(|a - a|),$$

we say that $e^{2\pi i x \cdot a}$ and $e^{2\pi i x \cdot a'}$ are *ϕ-approximately orthogonal*.

This is a weakened statement of the mutual orthogonality of exponential functions on Ω. In this talk, we show that if Ω is the unit ball in dimension $d > 1$, and ϕ decays sufficiently fast as $t \to \infty$, then the unit ball can not admit any ϕ-approximate orthogonal frame of exponentials. We'll wrap up the talk by announcing a new result that shows that even approximate average orthogonality is an impediment to the existence of an exponential frame.

Harmonic and Spectral Analysis
International Zoom Conference
May 31 – June 2, 2020

Symmetry problems in PDE

Alexander Ramm
Kansas State University

The results of this talk are published in [1]–[6]. These results include:

a) A complete solution of the Pompeiu problem [1], [2],

b) Proof of the Schiffer's conjecture [1],

c) Symmetry results for the Helmholtz equation [1],[3],

d) First symmetry results in harmonic analysis [4].

The method I have developed allows one to give a short proof to some old results [5] and get some geometrical results [6]. Let us formulate one of the basic results, see [1], [3]. Assume that

$$\begin{aligned} (\Delta + k^2)u &= c_0 \quad \text{in } D \\ u|_S &= c_1 \\ u_N|_S &= c_2 \end{aligned} \tag{1}$$

Here $D \subset \mathbb{R}^3$ is a bounded domain with a smooth boundary S, N is the outer unit normal to S, c_j, $j = 0, 1, 2$, are constants, $k^2 = \text{const.} > 0$.

Theorem 1. *If problem (1) has a solution and* $\left|c_1 - c_0 k^{-2}\right| + |c_2| > 0$*, then D is a ball of radius a, where a solves the equation* $j_0'(ka) = \frac{c_2}{kC_1} j_0(ka)$*,* $C_1 := c_1 - c_0 k^{-2}$.

References

[1] A. G. Ramm, Symmetry problems. The Navier–Stokes problem, *Morgan & Claypool Publishers*, 2019.

[2] A. G. Ramm, *Solution to the Pompeiu problem and the related symmetry problem*, Appl. Math. Lett., **63**, (2017), 28–33.

[3] A. G. Ramm, *Symmetry problems for the Helmholtz equation*, Appl. Math. Lett., **96**, (2019), 122–125.

[4] A. G. Ramm, *Symmetry problems in harmonic analysis*, SeMA, **78**, N1, (2021), 155–158.

[5] A. G. Ramm, *Old symmetry problem revisited*, Open Journ. Math. Analysis, (OMA), **2**, N2, (2018), 89-92.

[6] A. G. Ramm, *Necessary and sufficient condition for a surface to be a sphere*, Open J.Math.Anal. (OMA), **2**, (2018), issue 2, 51–52. Open access: https://pisrt.org/psr-press/journals/oma/

Spectral measures on groups with one prime factor

RUXI SHI
Mathematical Institute of the Polish Academy of Sciences

A Borel probability measure μ on a locally compact group is called a spectral measure if there exists a countable subset of continuous group characters which forms an orthogonal basis of the Hilbert space $L^2(\mu)$. It is known that a spectral measure is either discrete, absolutely continuous or singular continuous. The characterization of absolutely-continuous spectral measures goes back to Fuglede (1974) who conjectured that a Borel set is spectral if and only if it tiles by translation. Even though the conjecture was disproved, it still remains open for certain groups. In this talk, I will focus on locally compact abelian groups with one prime factor, say p, for example, $\mathbb{Z}/p^n\mathbb{Z}$, $(\mathbb{Z}/p\mathbb{Z})^d$, $\mathbb{Q}_p$ etc. I will discuss the characterization of spectral measures on groups with one prime factor.

Some properties of weighted Orlicz algebras

Seyyed Mohammad Tabatabaie
University of Qom

Weighted Orlicz spaces are significant generalizations of usual Lebesgue spaces with applications in several branches of mathematics. Recently, A. Osançlıol and S. Öztop showed that if $L^{\Phi}_w(G) \subseteq L^1_w(G)$, then $L^{\Phi}_w(G)$ is a convolution algebra, where G is a locally compact group, Φ is a Young function and w is a weight. They also studied the existence of approximate identity of this weighted algebra and characterized its spectrum by generalized characters. In this talk we will give some other necessary or sufficient conditions for a (weighted) Orlicz space to be a convolution Banach algebra in the context of locally compact groups and hypergroups.

Local spectral synthesis from a ring- theoretic perspective

BETTINA WILKENS
University of Namibia

Let G be an Abelian group. In his 2014 paper [Acta Math. Hungar. 143 (2014), no. 2, 313—329], *M. Laczkovich* considered the property of local spectral synthesis on G; this holds if every variety on G is the closure of the linear span of the local polynomials it contains. Local polynomials are functions restricting to a polynomial on any finitely generated subgroup of G.
We provide an alternative proof of Lazckovich's result that there is a cardinal κ with $\aleph_0 < \kappa < 2^{\aleph_0}$ such that local spectral synthesis pertains on G if and only if the torsion free rank of G is less than κ. We consider subdirectly irreducible quotients of localisations of $\mathbb{C}G$ and use extension theory of commutative rings. Our result extends to a characterisation of "locally synthesizable" individual varieties. In conclusion, we discuss a somewhat esoteric question: In a world where the continuum hypothesis fails, what more can be said about κ?