Symmetry problems in PDE

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The results of this talk are published in [1]–[6]. These results include:

a) A complete solution of the Pompeiu problem [1], [2],

b) Proof of the Schiffer's conjecture [1],

c) Symmetry results for the Helmholtz equation [1],[3],

d) First symmetry results in harmonic analysis [4].

The method I have developed allows one to give a short proof to some old results [5] and get some geometrical results [6]. Let us formulate one of the basic results, see [1], [3]. Assume that

$$
\begin{aligned}
(\Delta + k^2)u &= c_0 \quad \text{in } D \\
u|_S &= c_1 \\
u_N|_S &= c_2
\end{aligned}
\tag{1}
$$

Here $D \subset \mathbb{R}^3$ is a bounded domain with a smooth boundary S, N is the outer unit normal to S, c_j, $j = 0, 1, 2$, are constants, $k^2 = \text{const.} > 0$.

Theorem 1. *If problem (1) has a solution and $|c_1 - c_0 k^{-2}| + |c_2| > 0$, then D is a ball of radius a, where a solves the equation $j_0'(ka) = \frac{c_2}{kC_1} j_0(ka)$, $C_1 := c_1 - c_0 k^{-2}$.*

References

1. A. G. Ramm, Symmetry problems. The Navier–Stokes problem, *Morgan & Claypool Publishers*, 2019.

2. A. G. Ramm, *Solution to the Pompeiu problem and the related symmetry problem*, Appl. Math. Lett., **63**, (2017), 28–33.

3. A. G. Ramm, *Symmetry problems for the Helmholtz equation*, Appl. Math. Lett., **96**, (2019), 122–125.

4. A. G. Ramm, *Symmetry problems in harmonic analysis*, SeMA, **78**, N1, (2021), 155–158.

5. A. G. Ramm, *Old symmetry problem revisited*, Open Journ. Math. Analysis, (OMA), **2**, N2, (2018), 89-92.

6. A. G. Ramm, *Necessary and sufficient condition for a surface to be a sphere*, Open J.Math.Anal. (OMA), **2**, (2018), issue 2, 51–52. Open access: https://pisrt.org/psr-press/journals/oma/