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The null set of a polytope, and the Pompeiu property for polytopes

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A bounded domain $P \subset \mathbb{R}^d$ has the Pompeiu property if the values $\int_{\sigma(P)} f(x)\mathrm{d}x$ over all rigid motions $\sigma \in M(d)$ uniquely determine $f \in \mathcal{C}(\mathbb{R}^d)$. The Fourier-Laplace transform of a set P is the funtion $\widehat{1}_P \colon \mathbb{C}^d \to \mathbb{C}$, $\widehat{1}_P(z) = \int_P e^{-2\pi i \langle x,z\rangle}\mathrm{d}x$ and the null set is $N(P) = \left\{z \in \mathbb{C}^d : \hat{1}_P(z) = 0\right\}$. A classical result from Brown, Schreiber, and Taylor (1973) states that a set P does not have the Pompeiu property if and only if there exists $\alpha \in \mathbb{C} \setminus \{0\}$ such that $\left\{z \in \mathbb{C}^d : z_1^2 + \cdots z_d = \alpha\right\}$ is contained in $N(P)$. In this work we show that the null set of a polytope does not contain (almost all) circles and hence have the Pompeiu property. The proof is relatively simple and uses properties of Bessel functions together with the Brion-Barvinok theorem, which gives a concrete formulation for the Fourier-Laplace transform of a polytope. arXiv preprint: 2104.01957.