Harmonic and Spectral Analysis

International Zoom Conference

June 8–10 2020

General Information

The conference is held with the aid of the Zoom video conferencing program from **June 8**, 2020 to **June 10, 2020**. The participation is free but it is subjected to registration.

All conference **talks** are given **with the help of** the program **Zoom**, which enables sharing screen and also a whiteboard. The duration of every talk is at most 20 minutes, which is followed by a discussion of at most 5 minutes. There are no breaks between the talks within a session, therefore the schedule of the individual talks is only approximative. Speakers cannot inherit time from the previous talk. The time saved by shorter talks can be devoted to problems and remarks at the end of the session. If you have special wishes concerning the schedule, you are welcome to consult the conference secretary, *Eszter Gselmann* at the e-mail address gselmann@science.unideb.hu.

The host of the conference is the Department of Analysis of the University of Debrecen and the Alfréd Rényi Institute of Mathematics. The Organizing Committee consists of:

Prof.	**László Székelyhidi**	(Chair of the Organizing Committee)
Dr.	**Eszter Gselmann**	(Scientific Secretary)
Dr.	**Gergely Kiss**	(Scientific Secretary)

You can find the list of invited speakers, the list of the registered participants, the program and the abstracts in this booklet. Your questions may help the Organizing Committee to improve organization, so do not hesitate to contact the conference secretaries, *Eszter Gselmann* (gselmann@science.unideb.hu) and *Gergely Kiss* (kigergo57@gmail.com).

The Organizing Committee would like to express its great appreciation to the **Alfréd Rényi Institute of Mathematics** that provided us Zoom access during the conference.

List of Invited Speakers

1. Fechner, Żywilla *(Łódź University of Technology, Łódź, Poland)*
   Title of the talk: Finite dimensional varieties on hypergroups
   E-mail: zfechner@gmail.com

2. Gát, *György (University of Debrecen, Debrecen, Hungary)*
   Title of the talk: Varying parameter Cesàro and Riesz means of Walsh-Fourier series
   E-mail: gat.gyorgy@science.unideb.hu

3. Grinberg, Eric *(University of Massachusetts Boston, Boston, USA)*
   Title of the talk: The Radon transform on Helgason spheres in Hermitian symmetric spaces of compact type
   E-mail: eric.grinberg@umb.edu

4. Ivković, Stefan *(Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia)*
   Title of the talk: The generalized spectra of operators over C^*-algebras
   E-mail: stefan.iv10@outlook.com

5. Laczkovich, Miklós *(Eötvös Loránd University, Budapest, Hungary)*
   Title of the talk: Vector valued polynomials, exponential polynomials and vector valued harmonic analysis
   E-mail: laczk@cs.elte.hu

6. Malikiosis, Romanos *(Aristotle University of Thessaloniki, Thessaloniki, Greece)*
   Title of the talk: Recent developments in the discrete Fuglede conjecture
   E-mail: rwmanos@gmail.com

7. Mayeli, Azita *(The Graduate Center and Queensborough of the City University of New York, New York, USA)*
   Title of the talk: An interplay between Gabor bases and Fuglede conjecture
   E-mail: amayeli@gc.cuny.edu

8. Öztop, Serap *(Istanbul University, Istanbul, Turkey)*
   Title of the talk: Twisted Orlicz algebras on locally compact groups
   E-mail: oztops@istanbul.edu.tr

9. Puls, Michael *(John Jay College of Criminal Justice, New York, USA)*
   Title of the talk: Sets of p-restriction and p-spectral synthesis
   E-mail: mpuls@jjay.cuny.edu

10. Ramm, Alexander G. *(Kansas State University, Manhattan, USA)*
    Title of the talk: Symmetry problems in harmonic analysis
    E-mail: ramm@ksu.edu

11. Sawano, Yoshihiro *(Chuo University, Tokyo, Japan)*
    Title of the talk: Morrey spaces
    E-mail: yoshihiro-sawano@celery.ocn.ne.jp

12. Shulman, Ekaterina *(Vologda State University, Vologda, Russia and University of Silesia in Katowice, Katowice, Poland)*
    Title of the talk: From almost invariant subspaces to group covering
    E-mail: shulmanka@gmail.com

13. Somlai, Gábor *(Eötvös Loránd University, Budapest, Hungary)*
    Title of the talk: Fuglede's conjecture for groups that are the product of at most 4 cyclic groups
    E-mail: zsomlei@gmail.com

14. Székelyhidi, László *(University of Debrecen, Debrecen, Hungary)*
    Title of the talk: Spectral synthesis on the affine group of the unitary group
    E-mail: lszekelyhidi@gmail.com

15. Vati, Kedumetse *(Shanghai Jiao Tong University, Shanghai, China)*
    Title of the talk: Moment functions on hypergroup joins
    E-mail: vatluv2017@sjtu.edu.cn

16. Voit, Michael *(Technische Universität Dortmund, Dortmund, Germany)*
    Title of the talk: Continuous association schemes and hypergroups
    E-mail: michael.voit@math.tu-dortmund.de

17. Wilkens, Bettina *(University of Namibia, Windhoek, Namibia)*
    Title of the talk: Tensor products of synthesizable modules and a question on varieties with spectral synthesis
    E-mail: bwlklk@gmail.com

List of Participants

1. Agbeko, Kwami *(University of Miskolc, Miskolc, Hungary)*
2. Almira, José María *(University of Murcia, Murcia, Spain)*
3. Blahota, István *(University of Nyíregyháza, Nyíregyháza, Hungary)*
4. Burai, Pál *(University of Debrecen, Debrecen, Hungary)*
5. Ebanks, Bruce *(Mississippi State University, Mississippi, USA)*
6. Fechner, Włodzimierz *(Łódź University of Technology, Łódź, Poland)*
7. Fallon, Thomas *(The Graduate Center at the City University of New York, New York, USA)*
8. Gehér, György Pál *(University of Reading, Reading, UK)*
9. Gilányi, Attila *(University of Debrecen, Debrecen, Hungary)*
10. Gselmann, Eszter *(University of Debrecen, Debrecen, Hungary)*
11. Kiss, Gergely *(Alfréd Rényi Institute of Mathematics, Budapest, Hungary)*
12. Kiss, Tibor *(University of Debrecen, Debrecen, Hungary)*
13. Kumar, Vishvesh *(Ghent University, Ghent, Belgium)*
14. Lam, Nguyen H. *(Memorial University of Newfoundland, Grenfell Campus, Corner Brook, Canada)*
15. Mattheus, Sam *(Vrije Universiteit Brussel, Brussels, Belgium)*
16. Páles, Zsolt *(University of Debrecen, Debrecen, Hungary)*
17. Révész, Szilárd *(Alfréd Rényi Institute of Mathematics, Budapest, Hungary)*
18. Ross, Kenneth A. *(University of Oregon, Eugene, Oregon, USA)*
19. Rozenblum, Grigori *(Chalmers University of Technology, Göteborg, Sweden)*
20. Rösler, Margit *(Paderborn University, Paderborn, Germany)*
21. Sablik, Maciej *(University of Silesia in Katowice, Katowice, Poland)*
22. Singh, Ajit Iqbal *(Indian National Science Academy, New Delhi, India)*
23. Szokol, Patrícia *(University of Debrecen, Debrecen, Hungary)*

24. Tseng, Jimmy *(University of Exeter, Exeter, United Kingdom)*
25. Tsarev, Sergey P. *(Siberian Federal University, Krasnoyarsk, Russia)*
26. Yao, Liding *(University of Wisconsin–Madison, Madison, USA)*
27. Varga, Adrienn *(University of Debrecen, Debrecen, Hungary)*
28. Vértesi, Péter *(Alfréd Rényi Institute of Mathematics, Budapest, Hungary)*
29. Weisz, Ferenc *(Eötvös Loránd University, Budapest, Hungary)*
30. Zakaria, Amr *(University of Debrecen, Debrecen, Hungary)*

Program

June 8, Monday

13^{45}–15^{00}	1st **Afternoon Session, Chairman: László Székelyhidi**
13^{45}–14^{00}	**Opening (László Székelyhidi)**
14^{00}–14^{20}	**Michael Voit**, *Continuous association schemes and hypergroups*
14^{30}–14^{50}	**Michael Puls**, *Sets of p-restriction and p-spectral synthesis*
15^{00}–15^{30}	**Coffee Break**
15^{30}–16^{25}	2nd **Afternoon Session, Chairman: Michael Puls**
15^{30}–15^{50}	**Eric Grinberg**, *The Radon transform on Helgason spheres in Hermitian symmetric spaces of compact type*
16^{00}–16^{20}	**Alexander G. Ramm**, *Symmetry problems in harmonic analysis*

June 9, Thuesday

9^{00}–9^{55}	1st **Morning Session, Chairperson: Żywilla Fechner**
9^{00}–9^{20}	**László Székelyhidi**, *Spectral synthesis on the affine group of the unitary group*
9^{30}–9^{50}	**Bettina Wilkens**, *Tensor products of synthesizable modules and a question on varieties with spectral synthesis*
10^{00}–10^{30}	**Coffee Break**
10^{30}–11^{25}	2nd **Morning Session, Chairperson: Eszter Gselmann**
10^{30}–10^{50}	**Żywilla Fechner**, *Finite dimensional varieties on hypergroups*
11^{00}–11^{20}	**Kedumetse Vati**, *Moment functions on hypergroup joins*
11^{30}–14^{00}	**Lunch**
14^{00}–14^{55}	1st **Afternoon Session, Chairman: Włodzimierz Fechner**
14^{00}–14^{20}	**Serap Öztop**, *Twisted Orlicz algebras on locally compact groups*
14^{30}–14^{50}	**Problems and Remarks**
15^{00}–15^{30}	**Coffee Break**
15^{30}–16^{25}	2nd **Afternoon Session, Chairman: Zsolt Páles**
15^{30}–15^{50}	**Miklós Laczkovich**, *Vector valued polynomials, exponential polynomials and vector valued harmonic analysis*
16^{00}–16^{20}	**Azita Mayeli**, *An interplay between Gabor bases and Fuglede conjecture*

June 10, Wednesday

9^{00}–9^{55}	1st **Morning Session, Chairman: István Blahota**
9^{00}–9^{20}	**Yoshihiro Sawano**, *Morrey spaces*
9^{30}–9^{50}	**György Gát**, *Varying parameter Cesàro and Riesz means of Walsh-Fourier series*
10^{00}–10^{30}	**Coffee Break**
10^{30}–11^{25}	2nd **Morning Session, Chairman: Maciej Sablik**
10^{30}–10^{50}	**Ekaterina Shulman**, *From almost invariant subspaces to group covering*
11^{00}–11^{20}	**Stefan Ivković**, *The generalized spectra of operators over C^*-algebras*
11^{30}–14^{00}	**Lunch**
14^{00}–14^{55}	1st **Afternoon Session, Chairman: Gergely Kiss**
14^{00}–14^{20}	**Romanos Malikiosis**, *Recent developments in the discrete Fuglede conjecture*
14^{30}–14^{50}	**Gábor Somlai**, *Fuglede's conjecture for groups that are the product of at most* 4 *cyclic groups*
15^{00}–15^{20}	**Coffee Break**
15^{30}–16^{20}	2nd **Afternoon Session, Chairman: László Székelyhidi**
15^{30}–15^{50}	**Problems and Remarks**
16^{00}–16^{20}	**Closing (László Székelyhidi)**

Abstracts

Finite dimensional varieties on hypergroups

ŻYWILLA FECHNER
Łódź University of Technology
(joint work with LÁSZLÓ SZÉKELYHIDI)

Let X be a hypergroup, K its compact subhypergroup and assume that (X, K) is a Gelfand pair. Connections between finite dimensional varieties and K-polynomials on X are discussed. It is shown that a K-variety on X is finite dimensional if and only if it is spanned by finitely many K-monomials. Next, finite dimensional varieties on affine groups over $\mathbb{R}^d$, where d is a positive integer are discussed. A complete description of those varieties using partial differential equations is given.

REFERENCES

[1] **Żywilla Fechner, László Székelyhidi**, *Finite Dimensional Varieties on Hypergroups* (submitted)

[2] **László Székelyhidi**, Harmonic and Spectral Analysis, *World Scientific*, 2014.

[3] **László Székelyhidi**, *Spherical spectral synthesis*, Acta Math. Hungar. **153** (2017) 120–142.

Varying parameter Cesàro and Riesz means of Walsh-Fourier series

György Gát
University of Debrecen

Let $\alpha = (\alpha_n)$ be a sequence of reals, where $0 \le \alpha_n \le 1$ for every $n \in \mathbb{N}$. Let $\hat{f}(k) := \int_0^1 f(x)\omega_k(x)dx$ be the kth Walsh-Fourier coefficient of the integrable function f and define the (C, α) (varying parameter Cesàro) means of Walsh-Fourier series of f as

$$\sigma_n^{\alpha_n} f := \frac{1}{A_n^{\alpha_n}} \sum_{j=0}^{n} A_{n-j}^{\alpha_n} \hat{f}(j)\omega_j,$$

where $A_n^\beta := \frac{(1+\beta)\cdots(n+\beta)}{n!}$ for parameter $\beta \in \mathbb{R} \setminus \{-1, -2, \dots\}$. It is well-known, that for $\alpha_n = 1$ (for all n) we have the Fejér means and the a.e. relation $\sigma_n^1 f \to f$. Meanwhile, for $\alpha_n = 0$ (for every n), $\sigma_n^0 f$ is the nth partial sum of the Walsh-Fourier series of function f for what there exists a negative result, i.e. an integrable function f such as $\sigma_n^0 f \to f$ nowhere. This varying parameter Cesàro means is introduced and firstly investigated by Akhobadze [1] in the case of the trigonometric system. He gave some approximation results corresponding continuous functions. But no pointwise convergence result was given.

In this talk we show some recent almost everywhere convergence and divergence results of means of the kind above with respect to Walsh-Fourier series of integrable functions.

Reference

[1] **Teimuraz Akhobadze**, *On the convergence of generalized Cesàro means of trigonometric Fourier series*, Acta Math. Hungar. **115** (2007), no. 1-2, 59–78.

The Radon transform on Helgason spheres in Hermitian symmetric spaces of compact type

ERIC GRINBERG
University of Massachusetts Boston

We consider generalizations of the Radon transform from Euclidean space to symmetric spaces, of compact type. The first example is the celebrated Funk transform, which integrates over great circles on the sphere. These transforms are generally attached to totally geodesic varieties of various types, include maximal flat submanifolds, maximally dimensional, and maximally curved submanifolds. S. Helgason, in a 1966 paper, showed that there are many of the latter, and that they are usually spheres, now called Helgason Spheres. The dimensions and manifestations of these maximally curved spheres vary widely across the realm of symmetric spaces. As a consequence, we do not expect a uniform treatment of Radon transform across all symmetric ambient space. However, if we restrict to symmetric spaces with special additional structure, e.g., Lie groups viewed symmetric spaces, or Hermitian symmetric spaces, then uniformity sets in. Already known is the uniform treatment of injectivity of the Radon transform on Helgason spheres in Lie groups. We provide evidence for a unified treatment of uniqueness and invertibility properties of the Radon transform in the Hermitian context.

The generalized spectra of operators over C^*-algebras

STEFAN IVKOVIĆ
(Mathematical Institute of the Serbian Academy of Sciences and Arts)

If A is a C^*-algebra and F is an A-linear, bounded operator on a Hilbert C^*-module over A, then the operators of the form $F - aI$, where a runs over A, give rise to new generalized spectra in A of the operator F. By considering these generalized spectra in C^*-algebras of operators over C^*-algebras instead of ordinary spectra in the field of complex numbers, we obtain generalizations of various results from the classical spectral theory of operators. More precisely, in the classical operator theory, there are several versions of spectra, related to special classes of operators (Fredholm, semi-Fredholm, upper/lower semi-Fredholm, etc.). Replacing these spectra by the generalized spectra in C^*-algebras of operators over C^*-algebras, we show that most relations between these spectra are still true for these generalized versions. The relation between these spectra of an operator and those of its compressions is also transferred to the case of Hilbert C^*-modules. In addition, we consider perturbations of the generalized spectra in C^*-algebras of 2 by 2 operator matrices over C^*-algebras and prove generalizations of various results from the classical spectral theory concerning perturbations of spectra of operator matrices. Finally, we consider also the isolated points of the ordinary spectrum of operators over C^*-algebras and prove a generalization of the classical theorem about Riesz points in the setting of semi-Fredholm operators over C^*-algebras.

Vector valued polynomials, exponential polynomials and vector valued harmonic analysis

MIKLÓS LACZKOVICH
Eötvös Loránd University

Let G be a topological Abelian semigroup with unit, and let E be a Banach space. We define, for functions mapping G into E, the classes of polynomials, exponential polynomials and some other relevant classes. We establish their connections with each other and find their representations in terms of the corresponding complex valued classes.

We also investigate spectral synthesis and analysis in the class $C(G, E)$ of continuous functions $f\colon G \to E$. We show that if G is an infinite and discrete Abelian group and E is a Banach space of infinite dimension, then even spectral analysis fails in $C(G, E)$. We also prove that if G is discrete, has finite torsion free rank and if E is a Banach space of finite dimension, then spectral synthesis holds in $C(G, E)$.

Recent developments in the discrete Fuglede conjecture

ROMANOS MALIKIOSIS
Aristotle University of Thessaloniki
(joint work with GERGELY KISS, GÁBOR SOMLAI and MÁTÉ VIZER)

Fuglede's conjecture was stated in 1974, and connects an analytic with a geometric property of a given bounded measurable subset $T \subset \mathbb{R}^d$. In particular, it states that T accepts a complete orthogonal basis of exponential functions if and only if it tiles $\mathbb{R}^d$ by translations. This conjecture has been largely disproved, originally by Tao in 2004, and then by Farkas, Kolountzakis, Matolcsi, Móra and Révész for $d \geq 3$. This has been achieved by lifting counterexamples of Fuglede's conjecture in finite Abelian groups to counterexamples in Euclidean spaces. Since then, the characterization of finite Abelian groups satisfying (the discrete version of) Fuglede's conjecture has developed an in- terest for its own sake. We will present the recent results pertaining to this problem, including those of the speaker, along with Kiss, Somlai, Vizer.

An interplay between Gabor bases and Fuglede conjecture

Azita Mayeli
The Graduate Center and Queensborough
The City University of New York
(joint work with Chun-Kit Lai)

The Fuglede Conjecture asserts that a bounded and measurable domain Ω in $\mathbb{R}^d$ tiles the entire space by countable many copies of its translations if and only if the Hilbert space $L^2(\Omega)$ has an orthogonal basis of exponentials. The conjecture has been disproved for dimension $d \geq 5$ by Terry Tao (2003) and later for $d \geq 3$ by other mathematicians. However, the conjecture holds true for special cases in all dimensions. In this talk, we show how the study of the Gabor bases problem can be related to the study of Fuglede Conjecture in general. More precisely, we assert that for a characteristic function $g := \chi_\Omega$ of a set Ω, the function g generates a Gabor bases for $L^2(\mathbb{R}^d)$ with respect to a countable Gabor spectrum if and only if the Fuglede Conjecture holds ture for Ω. We term our assertion *Fuglede-Gabor Problem* and prove that it is true for special cases of Gabor spectrums.

Twisted Orlicz algebras on locally compact groups

SERAP ÖZTOP
Istanbul University
(joint work with EBRAHIM SAMEI and VARVARA SHEPELSKA)

Let G be a locally compact group, Φ be a Young function, and denote by $L^{\Phi}(G)$ the associated Orlicz space. I will give an introduction to twisted Orlicz algebras. Our approach allows us to systematically and simultaneously study twisted convolutions as well as weighted spaces. We apply our method in the form of twisted Orlicz algebras on compactly generated groups of polynomial growth.

Sets of p-restriction and p-spectral synthesis

MICHAEL PULS
John Jay College of the City University of New York

Let E be a closed set in $\mathbb{R}^n$. Recall that E is a set of spectral synthesis if there is only one closed ideal in $L^1(\mathbb{R}^n)$ with zero set E. In this talk we will extend the concept of spectral synthesis to $L^p(\mathbb{R}^n)$ for closed sets E that have the p-restriction property for some $p \in (1, 2)$. If time permits we will also give a connection between sets E of p-restriction and the problem of determining if the translations of $f \in L^1(\mathbb{R}^n) \cap L^p(\mathbb{R}^n)$ span $L^p(\mathbb{R}^n)$ when E is the zero set of f.

Symmetry problems in harmonic analysis

ALEXANDER G. RAMM
Kansas State University

Symmetry problems in harmonic analysis are formulated and solved, probably, for the first time. One of these problems is equivalent to the refined Schiffer's conjecture which was recently solved by the author, see [1].

Let $k = const > 0$ be fixed, S^2 be the unit sphere in $\mathbb{R}^3$, D be a connected bounded domain with C^2-smooth boundary S, $j_0(r)$ be the spherical Bessel function.

The harmonic analysis symmetry problems are stated in the following theorems.

Theorem A. *Assume that* $\int_S e^{ik\beta\cdot s}ds = 0$ *for all* $\beta \in S^2$. *Then S is a sphere of radius a, where* $j_0(ka) = 0$.

Theorem B. *Assume that* $\int_D e^{ik\beta\cdot x}dx = 0$ *for all* $\beta \in S^2$. *Then D is a ball.*

REFERENCE

[1] **Alexander G. Ramm**, Symmetry problems. The Navier-Stokes problem, *Morgan& Claypool Publishers, San Rafael, California,* 2019. ISBN 9781681735078

Morrey spaces

YOSHIHIRO SAWANO
Chuo University

The goal of this lecture is to survey Morrey spaces. I will give their definition and some elementary properties. This will be an introduction of our book 'Introduction and Applications to Integral Operators and PDE's, Volume I and II'.

From almost invariant subspaces to group covering

Ekaterina Shulman
Vologda State University, Russia
Silesian University in Katowice, Poland

The theory of multivariable addition theorems of Levi-Civita's type on semigroups is related to the study of finite-dimensional subspaces that are "almost translation invariant". It leads to some general problems on subadditve mappings.

Given a semigroup G and a set Ω we call a map $F : G \mapsto 2^{\Omega}$ *subadditive* if

$$F(gh) \subset F(g) \cup F(h) \quad \text{for all } g, h \in G. \tag{1}$$

We are interested in the following question: *suppose that each $F(g)$ contains $\leq n$ elements, does it imply that all $F(g)$ are contained in a finite set? If yes, what can one say about its cardinality?*

In the talk we are going to consider reformulations of such problems into the language of group covering. In particular, the following result will be discussed.

For any covering $G = G_1 \cup G_2 \ldots \cup G_k$ of a group G by its proper subgroups, there is an element of G that is covered by fewer than $k/2$ of these subgroups.

Fuglede's conjecture for groups that are the product of at most 4 cyclic groups

Somlai Gábor
Eötvös Loránd University and Alfréd Rényi Institute of Mathematics
(joint work with Gergely Kiss, Romanos Diogenes Malikiosis and Máté Vizer)

A bounded measurable set $\Omega \subset \mathbb{R}^n$ is called *spectral* if $L^2(\Omega)$ has an orthogonal basis consisting of exponential functions. We say that Ω is a *tile* if there it has a tiling complement T such that almost every $x \in \mathbb{R}^n$ can uniquely be written as $x = \omega + t$, where $\omega \in \Omega$ and $t \in T$.

Fuglede conjectured that Ω is spectral if and only if it is a tile. The conjecture was first disproved by Tao, who defined a version of the conjecture for finite groups and found a counterexample using Hadamard matrices. It has been proved that the spectral-tile direction of Fuglede's conjecture holds in $\mathbb{R}$ if and only if it holds for every finite cyclic group.

We will present ideas of the proofs that Fuglede's conjecture holds for $\mathbb{Z}_{pqrs}$, $\mathbb{Z}_{p^2qr}$ and $\mathbb{Z}_p^2 \times \mathbb{Z}_q$, where p, q, r, s are different primes.

Spectral synthesis on the affine group of the unitary group

LÁSZLÓ SZÉKELYHIDI
University of Debrecen

In this talk we prove spherical spectral synthesis on the affine group of $U(d)$, i.e. on the semidirect product $\mathbb{C}^d \rtimes U(d)$. Spherical spectral synthesis means ordinary spectral synthesis on the double coset hypergroup $X = \mathbb{C}^d \rtimes U(d)$. The space of continuous complex valued functions on X can be identified with the space of continuous $U(d)$-invariant functions on $\mathbb{C}^d$, that is, with the space of continuous radial functions on $\mathbb{C}^d$. The hypergroup-translation on this function space is realized by the following $U(d)$-translation:

$$f \mapsto \int_{U(d)} f(x + k \cdot y)\, d\omega(k),$$

where ω is the normalized Haar measure on $U(d)$. Then $U(d)$-varieties on $\mathbb{C}^d$ are those linear spaces of continuous radial functions on $\mathbb{C}^d$ which are closed with respect to all $U(d)$-translations, and with respect to compact convergence. In [2], we studied the basic building blocks of spherical spectral synthesis on the affine group of $U(d)$ over $\mathbb{C}^d$: $U(d)$-spherical functions and $U(d)$-moment functions. Using the results in [2], we deduce that finite dimensional $U(d)$-varieties consist of linear combinations of $U(d)$-moment functions, consequently, all $U(d)$-moment functions span a dense subspace in each $U(d)$-variety. The same result holds for the affine group of $SU(d)$. From the latter case we infer a complex generalization of L. Schwartz's spectral synthesis theorem in [1], even in several variables.

REFERENCES

[1] **Laurent Schwartz**, *Théorie générale des fonctions moyenne-périodiques*, Ann. of Math., **48**, (1947), 857–929.

[2] **Żywilla Fechner and László Székelyhidi**, *Spherical and moment functions on the affine group of* SU(n), Acta Math. Hungar., **157(1)**, (2019), 10–26.

Moment functions on hypergroup joins

KEDUMETSE VATI
Shanghai Jiao Tong University
(joint work with LÁSZLÓ SZÉKELYHIDI)

Moment functions play a basic role in probability theory. A natural generalization can be defined on hypergroups which leads to the concept of generalized moment function sequences. In a former paper we studied some function classes on hypergroup joins which play a basic role in spectral synthesis. Moment functions are also important basic blocks of spectral synthesis. All these functions can be characterized by well-known functional equations. In this talk we describe generalized moment function sequences on hypergroup joins.

Continuous association schemes and hypergroups

MICHAEL VOIT
Technische Universität Dortmund

Let (G, H) be a Gelfand pair, i.e., H is a compact subgroup of a locally compact group G such that the double coset hypergroup $G//H$ is commutative. In this case there exits a dual positive product formula for most characters which follows from the fact that these characters on $G//H$ may be seen as positive definite kernels on G/H and the fact that here positive definiteness is preserved by pointwise products.

On the other hand, in combinatorics there is the notion of finite commutative association schemes which may be seen as finite-dimensional algebras generated by finitely many stochastic matrices. Typical examples appear via finite Gelfand pairs (G, H) with G/H as state space where the matrices correspond to the elements of $G//H$. Moreover, there exist examples of finite commutative association schemes beyond groups, and each finite commutative association scheme leads to a finite commutative hypergroup whose dual also carries a hypergroup structure.

We extend the notion of classical finite commutative association schemes to the possibly infinite case where we relax the definition slightly by skipping the integrality conditions in the classical definition. This leads to examples which are associated to discrete commutative hypergroups. Moreover, we propose a topological generalization by using a locally compact basis space X and a family of Markov-kernels on X indexed by some locally compact space D. These so-called commutative continuous association schemes also lead to commutative hypergroup structures on D which have many features of double coset hypergroups like dual positive product formulas.

On the other hand, we have some rigidity results in the compact case which say that for given spaces X, D there are only a few continuous association schemes. In particular, all finite, commutative continuous association schemes (without any integrality condition) are automatically classical finite commutative association schemes. In the noncompact case no such rigidity result is known.

Tensor products of synthesizable modules and a question on varieties with spectral synthesis

BETTINA WILKENS
University of Namibia

Let V and W be synthesizable varieties for the (discrete) abelian groups G and H, respectively. We present a short proof of the fact that the variety $V \otimes W$ is synthesizable. Whether the property of having spectral synthesis carries over is a much more complicated matter.
Let V be a variety for the discrete abelian group G, let M be a maximal ideal of $\mathbb{C}G$ and n a natural number. If spectral synthesis holds on V, there is an absolute bound on the dimension of the indecomposable finite-dimensional submodules of V annihilated by M^n. Does the converse hold?