An interplay between Gabor bases and Fuglede conjecture

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The Fuglede Conjecture asserts that a bounded and measurable domain Ω in $\mathbb{R}^d$ tiles the entire space by countable many copies of its translations if and only if the Hilbert space $L^2(\Omega)$ has an orthogonal basis of exponentials. The conjecture has been disproved for dimension $d \geq 5$ by Terry Tao (2003) and later for $d \geq 3$ by other mathematicians. However, the conjecture holds true for special cases in all dimensions. In this talk, we show how the study of the Gabor bases problem can be related to the study of Fuglede Conjecture in general. More precisely, we assert that for a characteristic function $g := \chi_\Omega$ of a set Ω, the function g generates a Gabor bases for $L^2(\mathbb{R}^d)$ with respect to a countable Gabor spectrum if and only if the Fuglede Conjecture holds ture for Ω. We term our assertion *Fuglede-Gabor Problem* and prove that it is true for special cases of Gabor spectrums.