

# HARMONIC AND SPECTRAL ANALYSIS

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## Continuous association schemes and hypergroups

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Let  $(G, H)$  be a Gelfand pair, i.e.,  $H$  is a compact subgroup of a locally compact group  $G$  such that the double coset hypergroup  $G//H$  is commutative. In this case there exists a dual positive product formula for most characters which follows from the fact that these characters on  $G//H$  may be seen as positive definite kernels on  $G/H$  and the fact that here positive definiteness is preserved by pointwise products.

On the other hand, in combinatorics there is the notion of finite commutative association schemes which may be seen as finite-dimensional algebras generated by finitely many stochastic matrices. Typical examples appear via finite Gelfand pairs  $(G, H)$  with  $G/H$  as state space where the matrices correspond to the elements of  $G//H$ . Moreover, there exist examples of finite commutative association schemes beyond groups, and each finite commutative association scheme leads to a finite commutative hypergroup whose dual also carries a hypergroup structure.

We extend the notion of classical finite commutative association schemes to the possibly infinite case where we relax the definition slightly by skipping the integrality conditions in the classical definition. This leads to examples which are associated to discrete commutative hypergroups. Moreover, we propose a topological generalization by using a locally compact basis space  $X$  and a family of Markov-kernels on  $X$  indexed by some locally compact space  $D$ . These so-called commutative continuous association schemes also lead to commutative hypergroup structures on  $D$  which have many features of double coset hypergroups like dual positive product formulas.

On the other hand, we have some rigidity results in the compact case which say that for given spaces  $X, D$  there are only a few continuous association schemes. In particular, all finite, commutative continuous association schemes (without any integrality condition) are automatically classical finite commutative association schemes. In the noncompact case no such rigidity result is known.