Spectral synthesis on the affine group of the unitary group

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In this talk we prove spherical spectral synthesis on the affine group of $U(d)$, i.e. on the semidirect product $\mathbb{C}^d \rtimes U(d)$. Spherical spectral synthesis means ordinary spectral synthesis on the double coset hypergroup $X = \mathbb{C}^d \rtimes U(d)$. The space of continuous complex valued functions on X can be identified with the space of continuous $U(d)$-invariant functions on $\mathbb{C}^d$, that is, with the space of continuous radial functions on $\mathbb{C}^d$. The hypergroup-translation on this function space is realized by the following $U(d)$-translation:

$$f \mapsto \int_{U(d)} f(x + k \cdot y)\, d\omega(k),$$

where ω is the normalized Haar measure on $U(d)$. Then $U(d)$-varieties on $\mathbb{C}^d$ are those linear spaces of continuous radial functions on $\mathbb{C}^d$ which are closed with respect to all $U(d)$-translations, and with respect to compact convergence. In [2], we studied the basic building blocks of spherical spectral synthesis on the affine group of $U(d)$ over $\mathbb{C}^d$: $U(d)$-spherical functions and $U(d)$-moment functions. Using the results in [2], we deduce that finite dimensional $U(d)$-varieties consist of linear combinations of $U(d)$-moment functions, consequently, all $U(d)$-moment functions span a dense subspace in each $U(d)$-variety. The same result holds for the affine group of $SU(d)$. From the latter case we infer a complex generalization of L. Schwartz's spectral synthesis theorem in [1], even in several variables.

REFERENCES

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