Varying parameter Cesàro and Riesz means of Walsh-Fourier series

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Let $\alpha = (\alpha_n)$ be a sequence of reals, where $0 \leq \alpha_n \leq 1$ for every $n \in \mathbb{N}$. Let $\hat{f}(k) := \int_0^1 f(x)\omega_k(x)dx$ be the kth Walsh-Fourier coefficient of the integrable function f and define the (C, α) (varying parameter Cesàro) means of Walsh-Fourier series of f as

$$\sigma_n^{\alpha_n} f := \frac{1}{A_n^{\alpha_n}} \sum_{j=0}^{n} A_{n-j}^{\alpha_n} \hat{f}(j)\omega_j,$$

where $A_n^{\beta} := \frac{(1+\beta)\cdots(n+\beta)}{n!}$ for parameter $\beta \in \mathbb{R} \setminus \{-1, -2, \dots\}$. It is well-known, that for $\alpha_n = 1$ (for all n) we have the Fejér means and the a.e. relation $\sigma_n^1 f \to f$. Meanwhile, for $\alpha_n = 0$ (for every n), $\sigma_n^0 f$ is the nth partial sum of the Walsh-Fourier series of function f for what there exists a negative result, i.e. an integrable function f such as $\sigma_n^0 f \to f$ nowhere. This varying parameter Cesàro means is introduced and firstly investigated by Akhobadze [1] in the case of the trigonometric system. He gave some approximation results corresponding continuous functions. But no pointwise convergence result was given.

In this talk we show some recent almost everywhere convergence and divergence results of means of the kind above with respect to Walsh-Fourier series of integrable functions.

Reference

[1] **Teimuraz Akhobadze**, *On the convergence of generalized Cesàro means of trigonometric Fourier series*, Acta Math. Hungar. **115** (2007), no. 1-2, 59–78.